## Homework 3

P3.1.13 Determine $G_{e q}$ between terminals ' $a b$ ' in Figure P3.1.13, where $G$ is a conductance.
Solution: $G$ in series with $\frac{3}{2} G$ is $\frac{1 \times 1.5}{2.5}=0.6 G$. Two $0.6 G$ resistors are in parallel between the top and bottom nodes, giving a conductance of $1.2 G$. This resistor is in series with the $\frac{3}{2} G \frac{1.5 \times 1.2}{2.7} G=\frac{1.8 G}{2.7}=\frac{2 G}{3} ; G_{a b}=\frac{G}{2}+\frac{2 G}{3}=\frac{7}{6} G$.


Figure P3.1.13

P3.1.15 Determine $R_{e q}$ between terminals 'ab' in Figure P3.1.15, assuming all resistances are $1 \Omega$.

Solution: $R_{3}$ in parallel with $R_{4}$ is $0.5 \Omega$; this in series with $R_{2}$ is $1.5 \Omega$; this in parallel


Figure P3.1.15


Figure P3.1.15-1

P3.1.20 Determine $R_{\text {in }}$ in Figure P3.1.20.
Solution: The current in $R / 2$ is $2 V / R$. From KVL around the mesh in the middle, starting at node ' $c$ ' and going CCW: $-V_{c a}-V_{T}+2 V_{T}=0$, which gives $V_{c a}=V_{T}$. The current through $R$ in the middle branch is $V_{T / R}$ directed upwards. From KVL around the outer loop, starting at node 'b' and going CCW: $-2 V_{X}+V_{X}-V_{T}=$ 0 , which gives $V_{x}=-V_{T}$. The current in $R$ on the RHS is $V_{T /} / R$ directed upwards. From KCL at node 'a': $I_{T}+V_{T} / R+V_{T} / R-2 V_{T} / R=0$. This makes $I_{T}=0$, so that $R_{\text {in }}=V_{T} / I_{T} \rightarrow \infty$.


Figure P3.1.20-1

P3.2.7 Determine $V_{X}$ and $I_{Y}$ in Figure P3.2.7.

## Solution:




Figure P3.2.7

Figure P3.2.7-1

$$
\begin{aligned}
& \frac{40 \times 120}{40+120}=\frac{120}{4}=30 \Omega, \frac{60 \times 30}{60+30}=\frac{60}{3}=20 \Omega, \frac{20 \times 80}{20+80}=\frac{80}{5}=16 \Omega . \\
& V_{x}=\frac{16}{16+30+4} \times 25=\frac{16}{50} \times 25=8 \mathrm{~V} . \\
& I_{a}=\frac{25}{16+30+4}=\frac{25}{50}=0.5 \mathrm{~A}, I_{y}=\frac{120}{120+40} \times 0.5=\frac{3}{4} \times 0.5=0.375 \mathrm{~A} .
\end{aligned}
$$

P3.2.21 Determine $V_{o}$ in Figure P3.2.21 using source transformation.
Solution: The 2 A and 3.6 A sources and their parallel source resistances are transformed to their equivalent voltage sources. The circuit reduces to a two-essential-node circuit can be analyzed by applying KCL. The circuit becomes as shown. The two voltage sources and


Figure P3.2.21 the two resistances can be combined to simplify the circuit further. The 24 V source in series with the $8 \Omega$ resistor can be transformed to its equivalent current source. The two $8 \Omega$ resistors are combined in parallel into a $4 \Omega$ resistor. The current in this resistor is $(1+3) \mathrm{A}$,
 so that $V_{O}=4(1+3)=16 \mathrm{~V}$.


P3.2.23 Determine $V_{o}$ in Figure P3.2.23.
Solution: The current source in parallel with $10 \Omega$ is transformed to avoltage source of 2.5 V in series with $10 \Omega$. In series with $5 \Omega$ this is $15 \Omega$. The 2.5 V source in series with $15 \Omega$ is transformed to a current source of $2.5 / 15=1 / 6$ A in parallel with $15 \Omega$. The circuit becomes as shown. It follows from current division that:


Figure P3.2.23


Figure P3.2.23-1
$I_{X}=\left(1 / 6-0.5 I_{X}\right) / 2$. Hence, $I_{X}=1 / 15 \mathrm{~A}$, which gives $V_{O}=2 / 3 \mathrm{~V}$.

P3.3.8 Determine $I_{x}$ in Figure P3.3.8.
Solution: Initialize. All given circuit parameters and variables are entered. The nodes are labelled.

Simplify. The circuit is in a simple enough form.

Deduce. From Ohm; law, $V_{b d}=21 \times \mathrm{V}$; From KVL around the mesh 'bcdb', $I_{c b}=2\left(1-I_{X}\right) / 3$; if a closed surface is drawn as shown, $l_{a d}=2\left(1-I_{X}\right) / 3$; from KVL around the loop 'abda', $2\left(1-I_{x}\right) \times 3 / 3-4-2 I_{x}=0$, or $I_{x}=$ - 0.5 A.

Figure P3.3.8


Figure P3.3.8-1

P3.3.12 Determine $V_{Y}$ in Figure P3.3.12.
Solution: Initialize. All given parameters and variables are entered. The nodes are labelled.
Simplify. The $30 \Omega$ and $20 \Omega$ resistors are combined into a $50 \Omega$ resistor. The $30 \Omega$ and $15 \Omega$ resistors are combined into a $45 \Omega$ resistor. The circuit is redrawn to show it more clearly as a


Figure P3.3.12 two-essential-node circuit.
Deduce. $V_{a b}=50 I_{x}$; The current in the 30 V and $45 \Omega$ branch is: $\frac{50 I_{X}-30}{45}=\frac{10 I_{X}-6}{9}$ A. $50 \Omega$

From KCL at node 'a': $10-I_{X}$
$+2=\frac{10 I_{X}-6}{9}$, or $108-9 I_{x}=$

$10 I_{x}-6$, or $19 I_{x}=114$, which gives: $I_{x}=6$ A. It follows that $V_{a b}=$
$50 \times 6=300 \mathrm{~V}=V_{Y}-40 \times 2+40 \times 6$, or $V_{Y}=300-160=140 \mathrm{~V}$.

